it moves along the logarithmic axis. Two other features of logarithmic graphs should be mentioned. First, there is no zero on a logarithmic scale, so logarithmic variables cannot be plotted at the zero level. Second, any straight line in a logarithmic, or semilogarithmic, graph represents a geometric progression, meaning numbers like 2,4,8,16,32, . . . that increase by a constant proportion. Any curve that does not plot as a straight line does not represent a geometric progression. (We will explain this point below.)

Logarithmic graphs are important for two reasons:

1. They conveniently present data that vary over a wide range. Figure 2 does precisely this: the death rate for disease C has a few values as small as 3, and the death rate for disease B has values as large as 200. With an arithmetic Y axis, we would face a dilemma in the choice of scale. If we should calibrate the Y axis for the large rates of disease B, then the small rates of disease C would nearly vanish into obscurity because they would fall into a thin band along the bottom of the chart. But if we should calibrate the Y axis for the small rates, then the Y axis would have to be extremely long to accommodate the large rates; indeed, a vast empty space would appear on the graph between the two plots. A logarithmic Y axis lets us avoid the dilemma. Logarithmic ruling spreads apart small values, so that small rates for disease C become distinct; yet large rates for disease B can also be satisfactorily plotted without requiring the Y axis to have an unwieldy length.

2. A logarithmic graph shows proportional, or percentage, changes in a variable. Neither the arithmetic amount of change nor the variable's value at any point is of major importance in typical logarithmic graphs. To explain how proportional changes can be deduced from a logarithmic graph we must go back to the point made above: geometric progressions, with constant percentage changes, plot as straight lines on a logarithmic scale. A straight line results because the logarithms of a geometric progression form an arithmetic progression. For example, the common logarithms of 10, 100, 1,000, 10,000, . . . are, respectively, 1, 2, 3, 4, . . ., an arithmetic sequence which forms a straight line on a regular arithmetic scale. And the common logarithms of 2, 4, 8, 16, 32, . . . likewise form another arithmetic sequence which would plot as a straight line. It is actually logarithms that are being plotted in logarithmic graphs; that is, the plot of actual data points on a logarithmic scale is equivalent to a plot of corresponding logarithms on an arithmetic scale. And that is why a logarithmic axis becomes increasingly compressed.

The conclusion from these ideas—we must hold it in mind always when looking at logarithmic plots—is that equal distances on a logarithmic scale represent equal percentage changes, whereas equal distances on an arithmetic scale represent equal numerical changes. And a corollary of this conclusion is that equal slopes (or degrees of slant) on two logarithmic plots indicate equal rates of percentage change. Applying these generalizations in Figure 2, we see that for disease C the mortality rate dropped 50% between 1900 and 1912, then halved itself again between 1912 and 1918. (Three dots are placed on the plot for disease C so that the reader may project them to the axes and verify the preceding sentence.) For disease B the mortality rate dropped 50% between 1900 and about 1922; but as of 1938, when the study of disease B ended, the halving had not been repeated. (The two dots on the plot for disease B should be projected to the axes exactly as were the three dots for disease C.) Comparing the general downward slopes of the two plots, we see that the rate of percentage change in mortality from disease B is less than the rate of percentage change in mortality from disease C.